Ch. 16: Random Variables

* A numeric value based on the outcome of an event

* We use capital letters to denote random variables (like X, Y, Z)

* Example: tossing a coin 3 times and recording the number of heads: X = 0, 1, 2, 3 ~~X=2.5~~

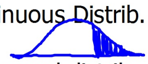
* Example: # of children in a family: X = 0, 1, 2, 3, ... ∞

~~X=2.5~~

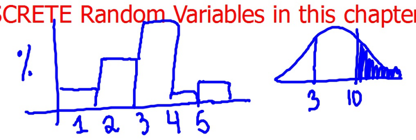
Random Variables: 2 types

Unit 1

	Discrete R.V. *	Continuous R.V.
What is it?	* Can list all outcomes	* Outcomes are in an interval * cannot list all
Examples:	* # of heads in 3 coin tosses * # of children in a family	(0, 2000) * # of hours a light bulb lasts * Time to run a mile (3:43, ∞) (4:13,)

	DISCRETE	CONTINUOUS
Distribution/ Function	* Probability Model X: 1, 2, 3, 4, 5 P(X): 0.1, 0.3, 0.5, 0.03, 0.07 =1 * Probability histogram $P(X=2)=0.30$	* Continuous Distrib.  * Ex: normal distrib., uniform distrib.
Properties of Distrib.	* Sum up to 1 $P(X<4)=0.90$ * All disjoint prob. Add prob. to get sums	* Area under curve = 1 $N(100, 5)$ * No individual probabilities!! $P(X\geq 10)=$

We will focus on DISCRETE Random Variables in this chapter



ample:

1) Let's play a game! You pay $5 to play. In the game, you get to draw one card from the deck.
   - If you draw the ace of hearts, you win $100
   - If you draw any other ace, you win $10
   - If you draw any other heart, you win $5
   - Any other card, you win nothing
   - We are interested in the amount that we GAIN

a) Create the probability distribution below:
   X = values of the variable. The outcomes
   P(X) = the probabilities of the values. Must sum to 1.

X	$95	$5	$0	-$5	
P(X)	1/52	3/52	12/52	36/52	= 1

b) What is the chance that you go home with at least some money? $P(X>0)=4/52$

c) What is the chance that you ~~win~~ GAIN at least $10? $P(X\geq 10)$ $P(X=95)$ $P(X\geq 95)$ $=1/52$ $P(X\geq 5)$ $P(X=5 \cup 95)$

d) What is the probability of not gaining any money? $P(X\leq 0)=48/52$

MEAN/STD DEVIATION:

MEAN = average gain (or average result) of one trial/playetc.

NOTATION: $\mu_X = E(X)$ weighted avg.

EXPECTED VALUE = MEAN = Average value we would expect to get if we played the game many, many times. AKA theoretical, long run average.

Formula for the mean (or expected value):

HW 10% × 95
CW 20% × 91
T 70% × 82 +

- X is a Discrete R.V. with the following distr.:

X	X1	X2	X3 ...	Xn
P(X)	P(X1)	P(X2)	P(X3) ...	P(Xn)

- Find the mean (expected value) by doing the following:

$$\sum X_i \cdot P(X_i)$$

CALCULATOR:

- L1 = values of variable (top row of table)
- L2 = Probabilities of each value (bottom row of table)
- STAT → CALC → 1-Var-Stats L1, L2
- Mean = $\bar{x}$ Std. Deviation = σ

Example:

X	0	1	2	3	4	5
P(X)	0.05	0.12	0.18	0.2	0.4	0.05

What is the mean (expected value) and the standard deviation?

$\mu_x = E(x) = 2.93 \quad \sigma_x = 1.306$

Example: Find the mean (expected value) and the std. deviation of the first game (card game).

x	\$95	\$5	\$0	-\$5
P(X)	1/52	3/52	12/52	36/52

$\mu_x = -\$1.346$

$\sigma_x = 13.802$

EXAMPLE: New game! We are rolling a standard die. We are again interested in the amount GAINED. It costs \$5 to play again.

- ** Roll a 6, you win \$10*
- ** Roll a 5, you win \$7*
- ** Roll a 3 or 4, you win \$5*
- ** Roll a 1 or 2, you win nothing*

Create the probability distribution, and then find the expected value and standard deviation of the game.

X	\$5	\$2	\$0	-\$5
P(X)	1/6	1/6	2/6	2/6

$\mu_x = -\$0.50$

$\sigma_x = \$3.594$

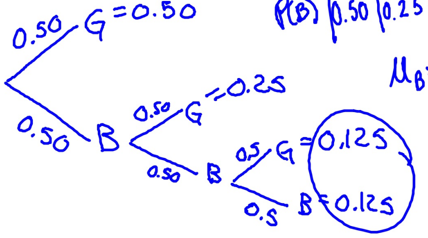
Book examples: p. 383 #5, 8, 23, ~~7~~

# Children	1	2	3
P(C)	0.50	0.25	0.25

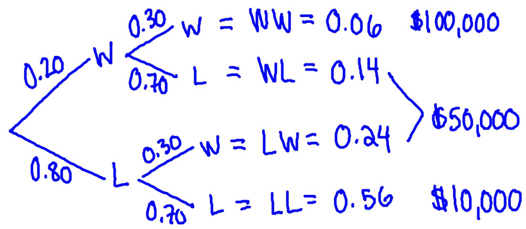
$E(C) = 1.75 = \mu_c$ children

B	0	1	2	3
P(B)	0.50	0.25	0.125	0.125

$\mu_B = E(B) = 0.875$ boys



(8)

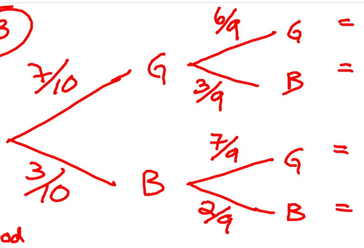


Profit	80,000	30,000	-10,000
P(P)	0.06	0.38	0.56

$\mu_p = E(P) = \$10,600$

$\sigma_p = \$25,877.40$

(23)



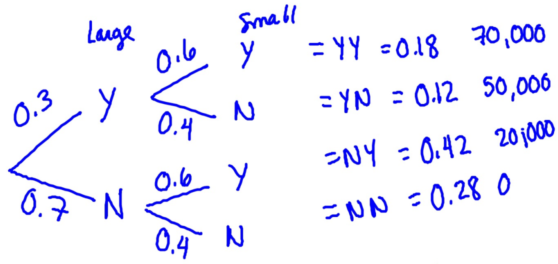
Good → X

X	0	1	2
P(X)	6/90	(21/90) + (21/90) = 42/90	42/90

E(X) = 1.4

SD(X) = 0.611

(7)



X = Profit	\$0	20,000	50,000	70,000
P(X)	0.28	0.42	0.12	0.18

EXAMPLE: You are playing a game that involves a spinner. When spun you will get 20 points 20% of the time, 5 points 40% of the time and 0 points the rest of the time.

a. Create the probability model for one spin.

b. Find the mean & standard deviation.

Pts	20	5	0
P(P)	0.20	0.40	0.40

$\mu_X = 6$ pts

$\sigma_X = 7.348$ pts.

c. Suppose the points were doubled. Find the new probability model, mean, & standard deviation.

d. Suppose you spun two times. Find the new probability model (use a tree diagram for help) and then the new mean & standard deviation for two spins.

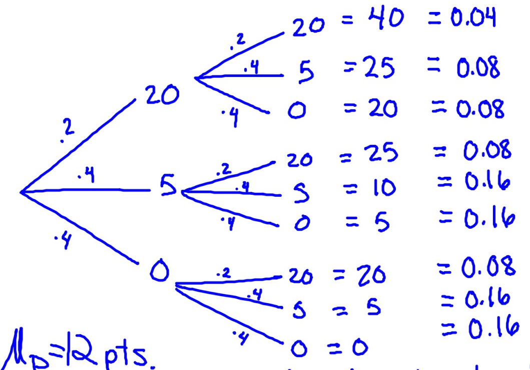
e. Are the probability models the same for Doubling points as they are for Spinning Twice?

f. Are the means the same between Doubling and Spinning Twice?

g. Are the standard deviations the same between Doubling and Spinning Twice?

P	40	10	0
P(P)	0.20	0.40	0.40

$\mu_X = 12$ pts $\sigma_X = 14.697$



$\mu_P = 12$ pts.

$\sigma_P = 10.392$

P	40	25	20	10	5	0
P(P)	0.04	0.16	0.16	0.16	0.32	0.16

Transforming Random Variables

* We may want to alter random variables (+, --, x, / the variable)
* We may want to combine it with another random variable too.

P + P

Let X and Y be INDEPENDENT random variables

Let "a" and "b" be fixed numbers

RULES: $a + bX$

MEAN: $\mu_{a+bX} = a + b(\mu_X)$

VARIANCE: $\sigma^2_{a+bX} = b^2\sigma_X^2$

$\sigma_{a+bX} = b\sigma_X$

REMINDER:
Std. Deviation = $\sqrt{variance}$
$\sigma = \sqrt{\sigma^2}$

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Std. Deviation = $\sqrt{variance}$
$\sigma = \sqrt{\sigma^2}$

(X and Y)

Combining 2 or more variables together:

MEAN: $\mu_{X+Y} = \mu_X + \mu_Y$

$\mu_{X-Y} = \mu_X - \mu_Y$

$\sigma_X = 7....$

$\sigma_X = 10...$

VARIANCE:

$\sqrt{\sigma^2_{X+Y}} = \sqrt{\sigma_X^2 + \sigma_Y^2}$

$(\sigma_Y)^2$

σ_Y^2

$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_X^2}$

σ_{X-Y}

EXAMPLES:

1)Random variable X has a mean of 6.2 and a std deviation of 3.1.

a. Find the new mean and standard deviation if we multiply by 3

$\mu_X = 6.2$ $\mu_{3X} = 18.6$

$\sigma_X = 3.1$ $\sigma_{3X} = 9.3$

b. Find the new mean and standard deviation if we subtract 10

$\mu_{X-10} = -3.8$

$\sigma_{X-10} = 3.1$

c. Find the new mean and standard deviation if we multiply by 5 and add 10.

$\mu_{5X+10} = 41$

$\sigma_{5X+10} = 15.5$

2) Random variable Y has a mean of 3.4 and a std deviation of 1.4.

$\mu_X = 6.2$

$\sigma_X = 3.1$

$\mu_Y = 3.4$

$\sigma_Y = 1.4$

a. Find the mean and standard deviation of X + Y

$\mu_{X+Y} = 9.6$

$\sigma_{X+Y} = \sqrt{(3.1^2) + (1.4^2)} = 3.401$

b. Find the mean and standard deviation of X – Y

$\mu_{X-Y} = 2.8$

$\sigma_{X-Y} = \sigma_{X+Y} = 3.401$

c. Find the mean and standard deviation of 2X + 3Y

$\mu_{2X+3Y} = 2(6.2) + 3(3.4) = 22.6$

$\sigma_{2X+3Y} = \sqrt{(2 \cdot 3.1)^2 + (3 \cdot 1.4)^2} = 7.4887$

d. Find the mean and standard deviation of 3X + Y – 4

$\mu_{3X+Y-4} = 3(6.2) + (3.4) - 4 = 18$

$\sigma_{3X+Y\cancel{-4}} = \sqrt{(3 \cdot 3.1)^2 + (1.4)^2} = 9.405$

3) New game! A single dice is rolled and the following occurs:

- Roll a 6, get 40 points
- Roll a 4 or 5, get 10 points
- Roll a 1, 2, or 3, get 0 points

a. Write the probability distribution

b. Find the mean and standard deviation of the game

c. Suppose the points are doubled. Find the new mean and standard deviation

d. Supposed the game is played twice (independently). What is the mean and standard deviation of this?

More examples:

1. Suppose that X is a random variable with $\mu_X = 10$ and $\sigma_X = 2$. Find the following:

   a. μ_{5X} b. μ_{X-3} c. μ_{5X-3}

   d. σ_{5X} e. σ_{X-3}

2. Suppose that X is the random variable from above, and that Y is independent from X with $\mu_Y = 15$ and $\sigma_Y = 3$. Find the following:

   a. μ_{X+Y} b. μ_{X+3Y} c. σ_{X+Y}

   d. σ_{X-Y} e. σ_{5X-2Y}

3. Suppose the mean SAT verbal score is 425 with standard deviation 100, while the mean SAT math score is 475 with standard deviation 100. What can be said about the mean and standard deviation of the combined math and verbal score?

Book examples: p. 384 #16 & 32, 24, 25, 38, 40

16) (a) $\mu_X = 2.25$ red lights
(b) $\sigma_X = 1.26$ red lights

32) $\mu_{X+X+X+X+X} = 11.25$ red lights
$\sigma_{X+X+X+X+X} = 2.82$ red lights

25) (a) $\mu = 30$, $\sigma = 6$ (b) $\mu = 26$, $\sigma = 5$ (c) $\mu = 30$, $\sigma = 5.39$ (d) $\mu = -10$, $\sigma = 5.39$

(e) $\mu = 20$
$\sigma = 2.83$

38) $\mu_D = 100$ $\mu_C = 120$
$\sigma_D = 30$ $\sigma_C = 35$

a) $\mu_{D-C} = 100 - 120 = -\$20$

b) $\sigma_{D-C} = \sqrt{30^2 + 35^2} = \$46.098$

c) $N(-20, 46.098)$

$P(D > C) = P(D - C > 0) = 33.22\%$ (subtract C from both sides)

normalcdf(0, E99, -20, 46.098)

40) a) $D + D + C$ = total cost

b) $\mu_{D+D+C} = 100 + 100 + 120 = \$320$

$\sigma_{D+D+C} = \sqrt{30^2 + 30^2 + 35^2} = \$55$

$N(320, 55)$

c) $P(\text{total} > 400) =$

= normalcdf(400, E99, 320, 55)

= 0.0729